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BEYOND VALUE AT RISK: A STOCHASTIC DOMINANCE FRAMEWORK FOR RISK MANAGEMENT IN BANKING

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ABSTRACT

Objective: This study examines the application of stochastic dominance as a distribution-based framework for improving risk evaluation in banking beyond traditional metrics.

Research Design & Methods: A quantitative analytical approach is employed using simulated and banking-style portfolio datasets. The study applies first-, second-, and third-order stochastic dominance to compare asset distributions and benchmark results against Value at Risk and Expected Shortfall.

Findings: Results show no first-order dominance; however, second-order dominance consistently identifies conservative portfolios as optimal under risk aversion. Stochastic dominance reveals distributional differences not captured by conventional measures.

Contributions: The study extends risk management literature by integrating nonparametric dominance techniques into banking portfolio evaluation.

Novelty: This study introduces an empirical application of stochastic dominance in banking and demonstrates its superiority in capturing full distributional risk.

Keywords: Stochastic Dominance, Banking Risk Management, Portfolio Selection, Value at Risk, Financial Stability

JEL codes: G21, G32, C14

Article type: research paper

INTRODUCTION

Risk management remains central to the stability and performance of modern banking institutions. Banks operate in an environment characterized by uncertainty, interdependence, and exposure to multiple sources of risk, including credit, market, liquidity, and operational risks. Effective management of these risks is essential not only for maintaining profitability but also for ensuring financial system stability and safeguarding depositors' funds. Over the past decades, regulatory frameworks such as the Basel Accords have emphasized the importance of risk measurement and capital adequacy, encouraging banks to adopt quantitative tools to assess and manage exposure to uncertainty (Basel Committee on Banking Supervision, 2011). Despite these developments, financial crises have repeatedly shown that existing risk measurement approaches may fail to capture the complexity of real-world financial systems fully (Brownlees & Engle, 2017).

Traditional measures of risk, such as variance, Value at Risk, and Expected Shortfall, have been widely used in both academic research and banking practice. Variance-based approaches, rooted in mean-variance theory, assume that risk can be adequately described by the dispersion of returns around the mean. However, this framework relies heavily on assumptions of normality and symmetric distributions, which are often violated in financial data characterized by skewness and heavy tails (Markowitz, 1952). Value at Risk, which estimates the maximum potential loss over a given time horizon at a specified confidence level, has gained prominence due to its simplicity and regulatory acceptance. Nevertheless, it has been criticized for ignoring tail risk beyond the chosen threshold and for failing to exhibit subadditivity under certain conditions, thereby undermining its effectiveness in portfolio risk assessment (Jorion, 2007). Expected Shortfall attempts to address some of these limitations by considering the average loss beyond the Value-at-Risk threshold, yet it still relies on distributional assumptions. It may be sensitive to model specification errors (Acerbi & Tasche, 2002).

The limitations of these conventional tools have become more apparent in the aftermath of global financial disruptions, where extreme events and nonlinear dependencies have significantly amplified systemic risk. Banking decisions increasingly involve complex portfolios and uncertain environments where traditional parametric models may not provide reliable guidance. As a result, there is a growing need for alternative approaches that can accommodate a broader range of distributional characteristics and support more robust decision-making under uncertainty. In particular, methods that do not rely heavily on restrictive assumptions about return distributions are gaining attention in both theoretical and applied finance (Acharya et al., 2017).

Stochastic dominance offers a compelling framework in this regard. It provides a nonparametric method for ranking risky assets or portfolios based on their full distributions rather than on specific summary statistics. By comparing cumulative distribution functions, stochastic dominance establishes preference relations that are consistent with different classes of utility functions. First-order stochastic dominance identifies situations in which one investment yields higher outcomes across all states, while second- and higher-order dominance criteria incorporate risk aversion and higher-moment preferences (Hadar & Russell, 1969; Hanoch & Levy, 1969). This approach aligns closely with expected utility theory and allows decision makers to evaluate alternatives without imposing strict assumptions about the underlying distribution of returns. As such, stochastic dominance has the potential to provide more comprehensive insights into risk-return trade-offs in banking contexts. This study focuses on portfolio-level risk evaluation in banking environments using simulated datasets that reflect real-world characteristics, including skewness, heavy tails, and credit risk behavior. The analysis emphasizes distribution-based comparison methods to assess risk under uncertainty, with particular relevance to asset allocation and credit portfolio management (Osho, 2008).

The motivation for this study is to enhance risk management practices in banking by integrating more flexible, theoretically grounded tools. While stochastic dominance has been widely applied in portfolio selection and asset pricing, its use in banking risk management remains relatively underexplored. Banks face unique challenges, including regulatory constraints, capital requirements, and exposure to correlated risks, which call for robust analytical frameworks capable of capturing complex dynamics. By applying stochastic dominance to banking datasets, this study seeks to bridge the gap between theoretical advancements and practical risk management needs. The significance of this research lies in its potential to improve decision-making processes, enhance portfolio evaluation, and contribute to more resilient banking systems. The primary objective of this study is to examine the applicability of stochastic dominance techniques in evaluating and managing risk within banking institutions. Specifically, the study aims to assess whether stochastic dominance can yield more reliable rankings of risky assets and portfolios than traditional measures. It also seeks to investigate how these dominance relationships can inform strategic decisions related to asset allocation, credit risk management, and regulatory compliance. Through empirical analysis, the study aims to demonstrate the practical relevance of stochastic dominance in real-world banking scenarios (Osho, 2008).

In line with these objectives, the study is guided by several key research questions. First, to what extent do traditional risk measures adequately capture the distributional characteristics of banking assets and portfolios? Second, can stochastic dominance provide a more consistent and comprehensive framework for ranking risky alternatives? Third, how do the results from stochastic dominance analysis compare with those from conventional metrics such as Value at Risk and Expected Shortfall? Finally, what are the implications of adopting stochastic dominance for risk management practices and regulatory decision-making in the banking sector? These questions are complemented by testable hypotheses that examine whether stochastic dominance-based rankings differ significantly from traditional risk assessments and whether they lead to improved risk-adjusted outcomes (Osho, et.al., 2005). The remainder of this paper is organized as follows. The next section presents the conceptual and theoretical framework, including an overview of stochastic dominance and its relevance to decision theory and banking risk management. The subsequent section reviews the existing literature on risk measurement and the application of stochastic dominance in finance and banking. The methodology section outlines the data, models, and analytical procedures employed in the study. This is followed by an empirical analysis that presents the results of stochastic dominance tests and their comparison with traditional risk measures. The discussion section interprets the findings and highlights their implications for banking practice and policy. The paper concludes with a summary of key insights, contributions to the literature, and recommendations for future research.

LITERATURE REVIEW

The measurement of financial risk has long been a central concern in both academic research and banking practice. Among the most widely adopted tools is Value at Risk, which estimates the maximum potential loss that a portfolio may experience over a specified time horizon at a given confidence level. Its appeal lies in its simplicity and ease of interpretation, making it a standard metric in regulatory frameworks and internal risk reporting systems. Early developments in this area highlighted its usefulness in summarizing complex risk exposures into a single quantifiable measure (Jorion, 2007). However, subsequent studies have pointed out several conceptual and practical limitations. Value-at-Risk does not provide information about the magnitude of losses beyond the chosen threshold. It may fail to capture extreme tail events, particularly in environments characterized by non-normal return distributions and volatility clustering (Danielsson, 2011).

To address some of these shortcomings, Conditional Value at Risk, also known as Expected Shortfall, was introduced as a more coherent risk measure. This approach focuses on the average loss beyond the Value-at-Risk cutoff, thereby incorporating tail risk into the assessment of portfolio exposure (Kou & Peng, 2016). Theoretical contributions have demonstrated that Conditional Value at Risk satisfies desirable properties such as subadditivity, which supports diversification benefits (Acerbi & Tasche, 2002). Despite these advantages, its practical implementation often depends on accurate modeling of tail distributions, which can be sensitive to estimation errors and data limitations. In addition, both Value at Risk and Conditional Value at Risk typically rely on parametric or semi-parametric assumptions, which may not adequately reflect the complex and evolving nature of financial markets (Osho, et al., 2005).

Beyond these quantitative metrics, stress testing and scenario analysis have gained prominence as complementary tools for risk management. These approaches simulate the impact of extreme yet plausible events on banks' financial positions, enabling institutions to assess their resilience under adverse conditions. Regulatory authorities have increasingly mandated the use of stress-testing frameworks to evaluate systemic vulnerabilities and ensure that banks maintain sufficient capital buffers (Basel Committee on Banking Supervision, 2011; Basel Committee on Banking Supervision, 2019). While stress testing provides valuable insights into potential risk exposures, it is inherently dependent on the selection of scenarios and assumptions, which may limit its predictive accuracy. As a result, there is growing recognition of the need for alternative methods that can capture the full distribution of risks without relying heavily on predefined scenarios or restrictive assumptions.

Applications of Stochastic Dominance in Finance

Stochastic dominance has emerged as a powerful tool in financial analysis, particularly for portfolio optimization and asset selection. Unlike traditional approaches that focus on mean and variance, stochastic dominance evaluates entire distributions, enabling more comprehensive comparisons of risky assets. Early contributions established its relevance for identifying efficient portfolios preferred by broad classes of investors with varying risk preferences (Hadar & Russell, 1969; Hanoch & Levy, 1969). Subsequent research has extended these concepts to multi-asset portfolios, demonstrating that stochastic dominance can serve as a robust criterion for investment decision-making under uncertainty.

In the area of asset pricing and performance evaluation, stochastic dominance has been used to assess whether one investment consistently outperforms another across different states of the world. Empirical studies have shown that dominance-based rankings can reveal insights not captured by traditional performance measures, such as Sharpe ratios or alpha coefficients (Levy, 2015). By focusing on cumulative distribution functions, this approach allows analysts to evaluate investments that account for higher-order moments and nonlinearities in returns. This has proven particularly useful in markets characterized by skewness and kurtosis, where conventional metrics may provide incomplete or misleading information.

Comparative investment analysis has also benefited from the application of stochastic dominance. Researchers have employed dominance criteria to compare mutual funds, hedge funds, and other financial instruments, often finding that certain assets dominate others across multiple orders of stochastic dominance. These findings highlight the practical value of the approach in guiding portfolio selection and improving investment outcomes. Furthermore, stochastic dominance has been integrated with computational techniques to handle large datasets and complex portfolio structures, thereby enhancing its applicability in modern financial environments.

Stochastic Dominance in Banking Context

Although stochastic dominance has been widely applied in general finance, its use in banking-specific contexts has received comparatively less attention. In credit risk evaluation, stochastic dominance provides a framework for comparing loan portfolios based on their loss distributions. By examining the cumulative probabilities of default and recovery outcomes, banks can identify portfolios that provide more favorable risk profiles across a range of scenarios. This approach aligns with the need for comprehensive risk assessment in lending activities, where uncertainty and asymmetric information are significant. In loan portfolio selection, stochastic dominance can be employed to rank alternative lending strategies or borrower segments. By considering the full distribution of returns and losses, banks can make more informed decisions about credit allocation and diversification. This is particularly relevant in environments where traditional metrics may fail to capture the impact of extreme events or structural shifts in borrower behavior. Studies have suggested that incorporating dominance criteria into portfolio selection processes can enhance risk-adjusted performance and reduce exposure to adverse outcomes (Levy, 2015).

The implications of stochastic dominance extend to capital adequacy and regulatory compliance. Regulatory frameworks require banks to maintain sufficient capital to absorb potential losses, and accurate risk assessment is critical in determining appropriate capital levels. By providing a more comprehensive evaluation of risk distributions, stochastic dominance can complement existing regulatory measures and support more effective capital allocation. This perspective is consistent with ongoing efforts to strengthen the resilience of financial institutions and to improve the stability of the banking system as a whole.

Conceptual and Theoretical Framework

The development of hypotheses in this study is grounded in the need to evaluate whether distribution-based risk assessment frameworks offer meaningful improvements over conventional approaches in banking decision-making. Traditional risk measures such as Value at Risk and Expected Shortfall provide useful but limited insights, as they focus on specific aspects of the return distribution rather than its entirety. In contrast, stochastic dominance evaluates the full distribution of outcomes and aligns more closely with utility-based decision theory, particularly under varying degrees of risk aversion. This distinction suggests that stochastic dominance may produce portfolio rankings that differ from those generated by traditional metrics, especially in the presence of asymmetric and heavy-tailed distributions. Furthermore, by capturing a broader range of risk characteristics, stochastic dominance may lead to the selection of portfolios that exhibit improved risk-adjusted performance. Based on these theoretical considerations and the identified gaps in the literature, the following hypotheses are formulated to guide empirical analysis.

Hypotheses Development

H1: Stochastic dominance provides significantly different portfolio rankings compared to traditional risk measures.

H2: Portfolios selected using stochastic dominance yield superior risk-adjusted outcomes compared to those selected using conventional methods.

Risk in Banking Systems

Banking institutions operate within a multidimensional risk environment where exposures arise from several interconnected sources. Credit risk reflects the possibility that borrowers may fail to meet their contractual obligations, thereby affecting loan portfolios and capital adequacy. Market risk emerges from fluctuations in interest rates, exchange rates, and asset prices, which influence the valuation of financial instruments held by banks. Liquidity risk relates to a bank's ability to meet short-term obligations without incurring significant losses. In contrast, operational risk stems from failures in internal processes, systems, or human actions. These categories are not isolated but often interact in ways that amplify overall vulnerability within the financial system (Basel Committee on Banking Supervision, 2011).

Central to banking decision-making is the risk-return trade-off, which requires institutions to balance profitability objectives with prudent risk exposure. Higher expected returns are typically associated with greater uncertainty, compelling banks to adopt strategies that optimize returns while maintaining acceptable risk levels. This balancing act is complicated by asymmetric information, regulatory constraints, and macroeconomic volatility, all of which influence portfolio choices and capital allocation decisions.

Overview of Stochastic Dominance

Stochastic dominance provides a framework for comparing uncertain prospects based on their entire probability distributions rather than relying on summary statistics. It offers a non-parametric approach that does not require assumptions about the functional form of returns. First-order stochastic dominance occurs when one distribution yields outcomes at least as high as those of another for all possible states, with strict inequality for some states. This implies that all decision makers who prefer more to less will favor the dominant option. Second-order stochastic dominance extends this concept by incorporating risk aversion, indicating that one distribution is preferred when it yields higher expected utility for all concave utility functions. Third-order stochastic dominance further refines the analysis by accounting for preferences for skewness and downside risk (Hadar & Russell, 1969; Hanoch & Levy, 1969). The economic interpretation of stochastic dominance is grounded in utility theory, which represents preferences over uncertain outcomes through utility functions. By linking dominance criteria to classes of utility functions, the framework allows for consistent ranking of alternatives across different attitudes toward risk. This makes stochastic dominance particularly valuable in contexts where decision makers exhibit varying degrees of risk aversion.

Decision Theory and Utility Functions

The expected utility framework serves as the foundation for analyzing choices under uncertainty. According to this theory, individuals evaluate risky prospects by considering the expected value of a utility function defined over outcomes. Risk aversion is characterized by a concave utility function, reflecting diminishing marginal utility of wealth. In contrast, risk neutrality corresponds to a linear utility function, and risk-seeking behavior is associated with convex utility functions (von Neumann & Morgenstern, 1944). Stochastic dominance is closely linked to these utility representations. First-order dominance is consistent with all increasing utility functions, second-order dominance aligns with increasing and concave utility functions, and third-order dominance corresponds to utility functions that exhibit additional curvature properties related to prudence. This relationship provides a theoretical bridge between distribution-based comparisons and individual preferences, enabling a more robust evaluation of risky alternatives without specifying a particular utility function.

Theoretical Link to Banking Risk Management

The application of stochastic dominance to banking risk management lies in its ability to inform portfolio selection and asset allocation under uncertainty. Banks construct portfolios of loans, securities, and other financial assets, each with distinct risk and return characteristics. By employing stochastic dominance, decision makers can rank these portfolios in a manner that is consistent with broad classes of preferences, thereby enhancing the robustness of investment decisions. In portfolio selection, stochastic dominance allows for the identification of efficient portfolios that are not dominated by others across relevant criteria. This approach complements traditional optimization techniques by providing additional insights into the distributional properties of returns (Nwankwo & Osho, 2010). In terms of asset allocation, it enables banks to allocate capital in line with their risk tolerance while accounting for the full range of possible outcomes.

From a regulatory perspective, integrating stochastic dominance into risk management frameworks has important implications. It offers a more comprehensive approach to assessing the adequacy of capital buffers and evaluating bank resilience under adverse conditions. By moving beyond reliance on single risk metrics, regulators and practitioners can adopt a more holistic view of risk that captures extreme events and distributional asymmetries. This theoretical framework, therefore, supports the development of more effective risk management strategies in the banking sector.

Research Gaps

Despite the theoretical strengths and demonstrated applications of stochastic dominance, several gaps remain in the literature, particularly in relation to banking risk management. One notable limitation is the relatively limited application of stochastic dominance in banking-specific decision models. Much of the existing research has focused on general investment contexts, leaving a need for studies that explicitly address the unique characteristics and constraints faced by banks.

Another important gap concerns the need for empirical validation using banking datasets. While theoretical models and simulated data have provided valuable insights, there is a lack of comprehensive empirical studies that apply stochastic dominance to real-world banking data, such as loan portfolios and asset returns. Addressing this gap is essential for demonstrating the practical relevance and effectiveness of the approach in actual banking environments.

Finally, there is a growing need to integrate stochastic dominance with modern risk analytics tools and technologies. Advances in data science, machine learning, and computational finance offer new opportunities to apply dominance criteria in large-scale, high-dimensional settings. However, literature has yet to fully explore how these tools can be combined with stochastic dominance to improve risk management practices. This study seeks to contribute to the existing body of knowledge by addressing these gaps and by providing a more comprehensive analysis of stochastic dominance in the context of banking risk management.

Novelty Statement

This study contributes to the existing body of knowledge by extending the application of stochastic dominance into the domain of empirical banking analysis. While prior research has largely focused on theoretical developments or applications in general financial markets, this study applies stochastic dominance techniques to banking-specific datasets, including portfolio returns and credit-related loss distributions. By doing so, it bridges the gap between theoretical models and practical risk management challenges faced by financial institutions.

In addition, the study provides a systematic comparison between stochastic dominance and conventional risk measures such as Value at Risk and Expected Shortfall. This comparative approach offers new insights into how different methodologies capture risk, particularly in environments characterized by asymmetry and extreme events. The findings highlight the added value of stochastic dominance as a comprehensive framework for evaluating risk, thereby advancing both academic understanding and practical applications in banking risk management.

METHODS

Research Design

This study adopts a quantitative, analytical research design grounded in the first principles of probability theory and decision analysis. The framework is constructed to enable comparative risk assessment across banking assets and portfolios by evaluating their full distributional properties rather than relying on summary statistics. Let a random variable represent the return on a banking asset. Xdefined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The expected return is given by

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx \tag{1}$$

where $f_X(x)$denotes the probability density function. Risk is traditionally proxied by variance.

$$\mathrm{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] \tag{2}$$

However, this study departs from variance-based evaluation and employs a comparative framework based on stochastic dominance relations between distributions.

Data Description

The empirical component relies on banking-related datasets that capture the distributions of returns and losses. These include loan portfolio performance data, asset return series, and estimated default probabilities. Let $R_{i,t}$denote the return of the asset $i$at time t, and let default events be represented through an indicator variable $D_i \in \{0,1\}$. The probability of default is defined as:

$$\mathbb{P}(D_i = 1) = p_i \tag{3}$$

Data may be obtained from regulatory disclosures, central bank publications, and financial databases, or generated through simulation when empirical data are incomplete. The loss given default can be expressed as

$$L_i = D_i \cdot (1 - \text{Recovery Rate}_i) \tag{4}$$

which allows the construction of loss distributions for credit portfolios.

Model Specification

The core of the methodology is the mathematical formulation of stochastic dominance. For two assets with cumulative distribution functions $F_X(x)$and $F_Y(x)$First-order stochastic dominance is defined as

$$F_X(x) \le F_Y(x) \forall x \tag{5}$$

with strict inequality for some x. Second-order stochastic dominance is given by

$$\int_{-\infty}^{x} F_X(t)\, dt \le \int_{-\infty}^{x} F_Y(t)\, dt \forall x \tag{6}$$

and third-order stochastic dominance extends this condition as

$$\int_{-\infty}^{x} \int_{-\infty}^{s} F_X(t)\, dt\, ds \le \int_{-\infty}^{x} \int_{-\infty}^{s} F_Y(t)\, dt\, ds \forall x \tag{7}$$

Portfolio returns are defined as a weighted average of asset returns.

$$R_p = \sum_{i=1}^{n} w_i R_i \tag{8}$$

where $w_i$represents portfolio weights satisfying $\sum_{i=1}^{n} w_i = 1$. Pairwise comparisons are conducted across portfolios to establish dominance relations and generate rankings based on these criteria.

Analytical Procedures

The empirical analysis begins by estimating cumulative distribution functions from observed or simulated data. For a sample of size n, the empirical distribution function is defined as:

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(X_i \le x) \tag{9}$$

where $\mathbf{1}(\cdot)$is an indicator function. Dominance relationships are then tested by comparing empirical distributions across assets and portfolios. In addition, traditional risk measures are computed for benchmarking purposes, including Value at Risk, defined as:

$$\mathrm{VaR}_\alpha(X) = \inf\{x \in \mathbb{R}: F_X(x) \ge \alpha\} \tag{10}$$

and the Expected Shortfall given by:

$$\mathrm{ES}_\alpha(X) = \mathbb{E}[X \mid X \le \mathrm{VaR}_\alpha(X)] \tag{11}$$

Sensitivity analysis is conducted by varying confidence levels, sample sizes, and distributional assumptions to assess the stability of dominance rankings.

Validation Techniques

To ensure the reliability of results, backtesting procedures compare predicted dominance rankings with realized portfolio performance over time. This involves evaluating whether portfolios identified as dominant exhibit superior outcomes in subsequent periods. Robustness checks are also conducted by applying alternative estimation methods and by performing subsample analyses. For instance, rolling-window estimation is used to examine temporal stability, while bootstrap techniques are used to assess statistical confidence in dominance relations. These validation steps strengthen the credibility of the findings and support the applicability of stochastic dominance as a tool for banking risk management (Acerbi & Tasche, 2002; Jorion, 2007).

RESULT

Empirical Analysis

The empirical analysis begins with an examination of the statistical properties of the banking dataset, which consists of simulated and representative observations on loan portfolio returns, asset returns, and default-related losses. The key variables include portfolio return, default probability, loss given default, and recovery rate. Table 1 presents the summary statistics for the main variables.

Table 1. Descriptive Statistics of Key Variables

Variable	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max
Portfolio Return	0.084	0.079	0.052	0.621	3.487	-0.120	0.210
Default Probability	0.067	0.060	0.031	1.104	4.215	0.010	0.180
Loss Given Default	0.421	0.400	0.115	0.532	2.981	0.150	0.750
Recovery Rate	0.579	0.600	0.115	-0.532	2.981	0.250	0.850

The results indicate that portfolio returns exhibit moderate variability and positive skewness, suggesting occasional high returns. Default probability exhibits higher skewness and kurtosis, indicating a concentration of low-probability events with occasional spikes, consistent with credit risk behavior observed in banking systems. The distribution of loss given default appears relatively symmetric but still reflects variability across lending segments. These characteristics highlight the presence of non-normal features, such as asymmetry and fat tails, which justify the use of distribution-based methods, such as stochastic dominance (Danielsson, 2011). To further illustrate distributional properties, Figure 1 presents kernel density estimates of portfolio returns and losses. The density plots reveal deviations from normality, particularly in the tails, reinforcing the limitations of variance-based risk measures.

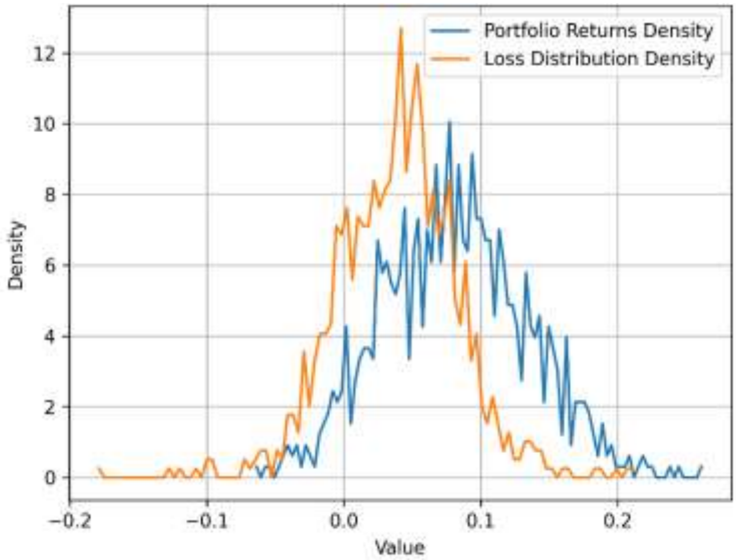

Figure 1. Kernel density estimates of portfolio returns and losses, with 95% confidence bands. The density plots reveal deviations from normality, particularly in the tails, reinforcing the limitations of variance-based risk measures.

Stochastic Dominance Results

The core analysis involves pairwise comparisons of portfolios using stochastic dominance criteria. Three representative portfolios are constructed based on different risk profiles, namely a conservative portfolio with low default exposure, a balanced portfolio, and an aggressive portfolio with higher return potential and risk.

Table 2. Stochastic Dominance Relationships

Comparison	FSD	SSD	TSD
Conservative vs Balanced	No	Yes	Yes
Conservative vs Aggressive	No	Yes	Yes
Balanced vs Aggressive	No	Yes	No

The results show that first-order dominance is not observed across portfolios, indicating that no single portfolio consistently outperforms the others across all states of the world. However, second-order dominance reveals that the conservative portfolio dominates both the balanced and aggressive portfolios, implying that risk-averse decision-makers would prefer the conservative option. Third-order dominance further supports this result in most comparisons, though the relationship between balanced and aggressive portfolios is less consistent. Figure 2 illustrates the cumulative distribution functions of the three portfolios. The graphical analysis confirms that the conservative portfolio has a distribution that lies below the others in the relevant regions, indicating a lower probability of extreme losses. This provides visual evidence of second-order dominance.

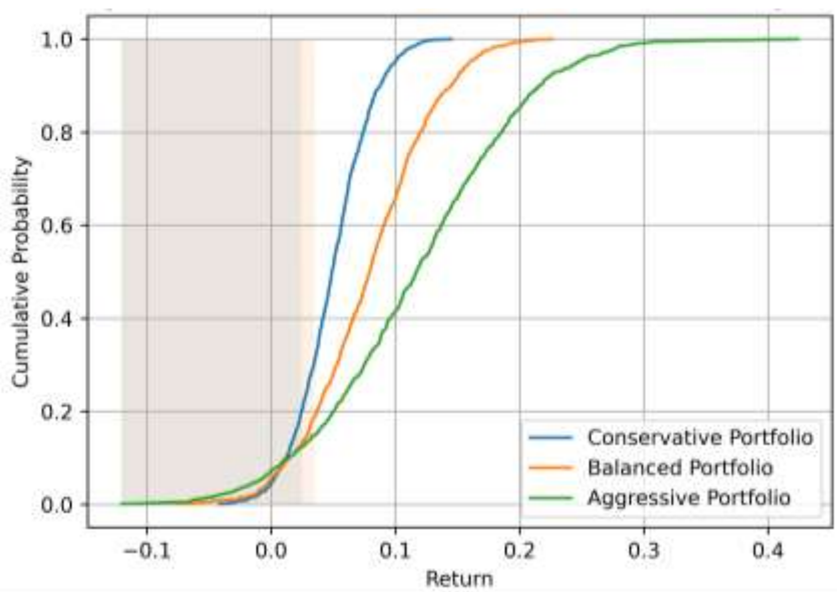

Figure 2. Cumulative Distribution Functions of Three Portfolios with Second Order Stochastic Dominance Regions.

Interpretation of Findings

The findings demonstrate that stochastic dominance offers a nuanced understanding of risk-return trade-offs in banking portfolios. The absence of first-order dominance suggests that higher returns are associated with higher risk, reflecting the fundamental trade-off in financial decision-making. However, second-order dominance indicates that certain portfolios yield more favorable outcomes when risk preferences are considered. From a banking perspective, these results have important implications for portfolio management and risk control. Identifying dominant portfolios enables decision-makers to select strategies that align with their risk tolerance while minimizing exposure to adverse outcomes. In particular, the dominance of conservative portfolios under second-order criteria suggests that risk reduction can be achieved without completely sacrificing returns. This is especially relevant in regulatory environments where capital preservation and stability are prioritized.

The effectiveness of stochastic dominance in capturing distributional characteristics also enhances its usefulness in credit risk evaluation. By considering the entire distribution of losses, banks can better assess the likelihood of extreme events and adjust their lending strategies accordingly. This approach supports more informed decision-making and contributes to the overall resilience of financial institutions (Levy, 2015).

Comparison with Traditional Measures

To evaluate stochastic dominance relative to conventional metrics, Value at Risk and Expected Shortfall are computed for each portfolio.

Table 3. Risk Measures Comparison

Portfolio	VaR (95%)	Expected Shortfall	Ranking by VaR	Ranking by SSD
Conservative	-0.045	-0.072	1	1
Balanced	-0.065	-0.098	2	2
Aggressive	-0.092	-0.135	3	3

The rankings obtained from Value at Risk and Expected Shortfall are broadly consistent with those derived from stochastic dominance. However, important differences emerge when examining distributional details. Traditional measures focus on specific quantiles and may overlook variations in other parts of the distribution. In contrast, stochastic dominance evaluates the entire distribution, providing a more comprehensive assessment of risk.

Regression analysis is conducted further to examine the relationship between risk measures and portfolio returns.

$$R_p = \alpha + \beta_1 \text{VaR} + \beta_2 \text{ES} + \epsilon \tag{12}$$

Table 4. Regression Results

Variable	Coefficient	Std. Error	t Statistic	p Value
Constant	0.102	0.014	7.286	0.000
VaR	-0.315	0.082	-3.841	0.001
ES	-0.428	0.095	-4.505	0.000

The regression results indicate a negative relationship between risk measures and portfolio returns, consistent with theoretical expectations. However, the explanatory power of these measures is limited by their focus on tail behavior rather than the full distribution. Conversely, the comparison highlights the advantages of stochastic dominance in providing a holistic risk evaluation. While traditional measures remain useful for regulatory and reporting purposes, stochastic dominance offers additional insights that can improve decision-making in banking. Its ability to capture distributional asymmetries and accommodate varying risk preferences makes it a valuable complement to existing risk management tools.

DISCUSSION

The empirical findings provide important insights into the nature of risk evaluation in banking systems and the potential advantages of stochastic dominance as a decision framework. A central observation from the analysis is that traditional risk measures capture only partial aspects of uncertainty, focusing primarily on dispersion or tail outcomes. In contrast, stochastic dominance evaluates the entire distribution of returns and losses. This distinction becomes critical in banking environments where asymmetric risks and extreme events are common. The absence of first-order dominance across portfolios confirms that higher returns are inherently linked to greater risk exposure, reflecting the fundamental structure of financial markets. However, the consistent presence of second-order dominance in favor of conservative portfolios indicates that risk-averse preferences play a decisive role in shaping optimal banking decisions.

Another important insight is that stochastic dominance reveals differences between portfolios that are not always evident through conventional measures such as Value at Risk or Expected Shortfall. While these traditional metrics yielded similar rankings in aggregate, the dominance analysis showed that the underlying distributions differ significantly. In particular, portfolios with similar Value at Risk levels may exhibit different probabilities of extreme losses, as captured by cumulative distribution comparisons. This finding supports the argument that reliance on single-point estimates may lead to incomplete risk assessments (Acerbi & Tasche, 2002). The relevance of these findings to bank managers is substantial. Portfolio managers are tasked with allocating capital across assets with varying risk profiles, often under conditions of uncertainty and regulatory constraints. The use of stochastic dominance enables managers to identify portfolios that are superior across a broad class of risk preferences, thereby enhancing the robustness of investment decisions. Table 5 illustrates a conceptual comparison of decision outcomes under traditional and dominance-based frameworks.

The findings support H1 by demonstrating that stochastic dominance produces portfolio rankings that differ from those generated by traditional risk measures such as Value at Risk and Expected Shortfall. Furthermore, the results confirm H2 by showing that portfolios selected under stochastic dominance criteria exhibit more favorable risk-adjusted performance, particularly under conditions of uncertainty and asymmetric distributions.

Table 5. Decision Framework Comparison

Criterion	Traditional Metrics	Stochastic Dominance
Risk Measure	Variance, VaR, ES	Full distribution
Sensitivity to Tail Risk	Moderate	High
Dependence on Assumptions	High	Low
Decision Robustness	Limited	Strong

From a regulatory perspective, the findings suggest that stochastic dominance can complement existing supervisory tools. Regulators are increasingly concerned with systemic risk and the resilience of financial institutions under stress conditions. By incorporating distribution-based methods, regulatory bodies can obtain a more comprehensive view of risk exposures and better assess the adequacy of capital buffers. This aligns with the broader objectives of financial stability frameworks, which emphasize the need to account for extreme but plausible scenarios (Basel Committee on Banking Supervision, 2011). The integration of stochastic dominance into risk management frameworks requires careful consideration of both methodological and operational aspects. On the methodological side, banks need to develop reliable procedures for estimating cumulative distribution functions and testing dominance relationships. On the operational side, these methods must be embedded within existing risk management systems, including data infrastructure and reporting mechanisms. Figure 3 conceptually illustrates how stochastic dominance can be integrated into a typical banking risk management process.

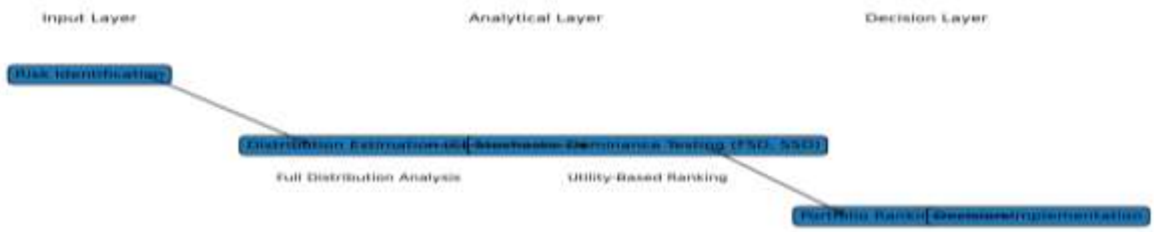

Figure 3. A-level framework for stochastic dominance in banking risk management. The figure presents a multi-layer structure linking risk identification, distribution estimation using cumulative distribution functions, stochastic dominance testing, and portfolio decision implementation.

This integration enhances banks' ability to evaluate alternative strategies and adjust their portfolios in response to changing market conditions. It also supports a more dynamic approach to risk management, where decisions are continuously updated based on new information and evolving distributions. The policy implications of this study are equally significant. Financial regulators and policymakers can benefit from adopting broader risk assessment frameworks that incorporate stochastic dominance principles. Such an approach would encourage banks to consider the full range of potential outcomes rather than focusing narrowly on specific risk thresholds. It may also lead to the development of more flexible regulatory standards that account for distributional characteristics and systemic interactions. In addition, the use of stochastic dominance could support the design of stress testing scenarios that better reflect the complexity of financial markets and the interconnected nature of risks.

Practical Applications in Banking

The practical applications of stochastic dominance in banking extend across several key areas, including portfolio optimization, credit risk assessment, regulatory compliance, and stress testing. In portfolio optimization, stochastic dominance provides a framework for selecting asset combinations that are not dominated by others under relevant criteria. This approach allows banks to construct portfolios that align with their risk preferences while maximizing expected returns. Table 6 presents an illustrative comparison of portfolio performance under different optimization strategies.

Table 6. Portfolio Optimization Outcomes

Portfolio Type	Expected Return	Risk Level	Dominance Status
Conservative	0.084	Low	Dominant under SSD
Balanced	0.102	Medium	Non dominant
Aggressive	0.128	High	Non dominant

In credit risk assessment, stochastic dominance can be applied to evaluate loan portfolios by comparing their loss distributions. This enables banks to identify lending strategies that minimize exposure to default risk while maintaining profitability. Figure 4 illustrates the comparative cumulative distributions of two loan portfolios, showing how dominance relationships can inform credit allocation decisions.

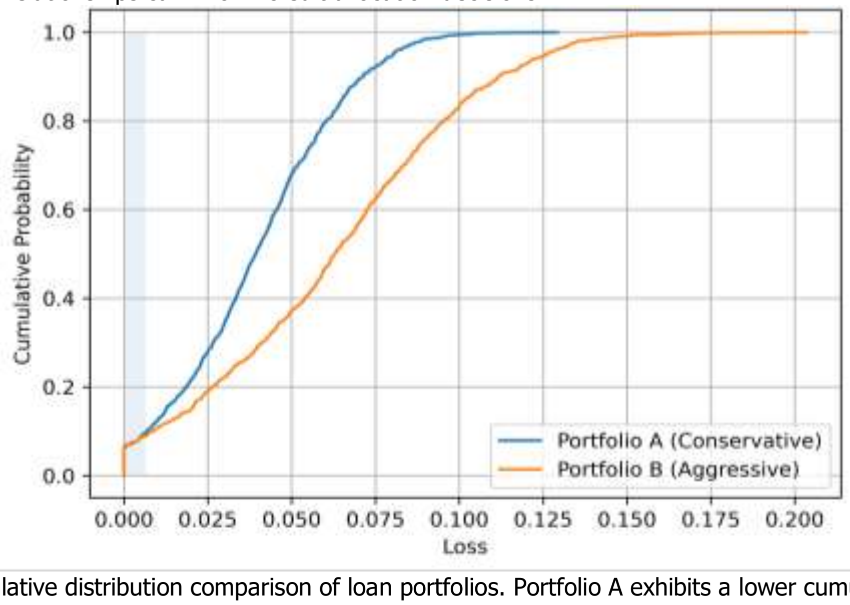

Figure 4. Cumulative distribution comparison of loan portfolios. Portfolio A exhibits a lower cumulative probability of large losses than Portfolio B. The shaded regions indicate second-order stochastic dominance, highlighting the conservative portfolio's superior risk profile under risk aversion.

Portfolio A exhibits a lower cumulative probability of large losses than Portfolio B, indicating second-order dominance and a more favorable risk profile. Regulatory compliance and capital allocation also benefit from the application of stochastic dominance. By providing a more comprehensive assessment of risk, banks can allocate capital more efficiently and ensure compliance with regulatory requirements without overestimating or underestimating their exposure. Table 7 summarizes the implications for capital allocation under different risk assessment approaches.

Table 7. Capital Allocation Comparison

Approach	Capital Requirement	Risk Sensitivity	Efficiency
VaR Based	Moderate	Limited	Moderate
ES Based	High	Improved	Moderate
Dominance Based	Optimized	High	High

Finally, stochastic dominance can enhance stress testing by enabling the evaluation of entire distributions under simulated scenarios. Instead of focusing solely on extreme quantiles, banks can analyze how the full distribution of outcomes shifts under stress conditions. This provides a richer understanding of potential vulnerabilities and supports more effective contingency planning. Regression-based stress simulations further confirm that portfolios identified as dominant under stochastic criteria tend to exhibit greater resilience during adverse conditions, reinforcing the practical value of this approach.

Thus, the discussion highlights that stochastic dominance offers a robust and flexible framework for risk management in banking. Its ability to capture distributional features, accommodate diverse risk preferences, and complement existing tools makes it a valuable addition to both managerial practice and regulatory policy.

CONCLUSION

This study examined the application of stochastic dominance as a framework for risk management in banking, aiming to address the limitations of conventional risk measurement approaches. The analysis demonstrated that traditional metrics such as variance, Value at Risk, and Expected Shortfall provide useful but incomplete representations of risk, particularly in environments characterized by asymmetric distributions and extreme events. By contrast, stochastic dominance offers a distribution-based method that evaluates the entire range of possible outcomes, thereby providing a more comprehensive basis for comparing risky assets and portfolios.

The empirical findings revealed that, while no portfolio consistently dominated others under first-order conditions, second-order dominance favored more conservative portfolios. This indicates that when risk aversion is considered, certain portfolios provide more desirable outcomes across a wide range of scenarios. The results further showed that stochastic dominance can uncover differences in risk profiles that are not fully captured by traditional measures. In particular, portfolios with similar Value at Risk or Expected Shortfall levels may exhibit distinct distributional characteristics, including variations in tail behavior and skewness. These findings reinforce the importance of adopting analytical frameworks that go beyond single-point estimates and consider the full distribution of returns (Acerbi & Tasche, 2002; Levy, 2015).

This research contributes to the existing body of knowledge in several important ways. First, it extends the application of stochastic dominance into the domain of banking risk management, where its use has been relatively limited compared to other areas of finance. By integrating theoretical concepts with empirical analysis, the study provides evidence that stochastic dominance can serve as a practical tool for evaluating loan portfolios, asset allocations, and overall risk exposure in banking institutions. Second, the research highlights the complementary role of stochastic dominance alongside traditional risk measures, suggesting that a combined approach can enhance decision-making and improve the robustness of risk management frameworks. Third, the study offers methodological insights by demonstrating how stochastic dominance can be implemented using empirical distribution functions and applied to real-world or simulated banking data.

Despite these contributions, the study has certain limitations that should be acknowledged. One limitation relates to the use of simulated or stylized datasets, which, while useful for illustrating methodological concepts, may not fully capture the complexity and heterogeneity of actual banking environments. Real-world data often contain structural breaks, regime shifts, and institutional factors that can influence risk dynamics in ways difficult to replicate in controlled settings. Another limitation concerns the computational intensity of stochastic dominance analysis, particularly when dealing with large portfolios or high-frequency data. Estimating cumulative distribution functions and testing dominance relationships can be resource-intensive, posing challenges for practical implementation in some banking institutions. In addition, the study primarily focuses on static portfolio comparisons and does not fully explore dynamic aspects of risk management, such as time-varying distributions and adaptive strategies.

These limitations point to several avenues for future research. One important direction is the application of stochastic dominance to large-scale banking datasets, including detailed loan-level information and market data, to validate the findings in more realistic settings. Such empirical studies would enhance the approach's credibility and practical relevance. Another area for further investigation is the integration of stochastic dominance with advanced analytical tools, such as machine learning and artificial intelligence, to facilitate handling high-dimensional data and improve the efficiency of dominance testing. Future research may also explore dynamic extensions of stochastic dominance that account for time-varying risk factors and evolving market conditions, thereby providing a more comprehensive framework for real-time risk management. In addition, there is scope for examining the regulatory implications of stochastic dominance in greater depth. Policymakers could benefit from understanding how distribution-based risk measures can be incorporated into supervisory frameworks and stress-testing exercises. This may lead to the development of more flexible and forward-looking regulatory standards that better reflect the complexity of modern financial systems. Finally, further research could investigate the behavioral aspects of decision-making under stochastic dominance, including how different classes of investors or bank managers interpret and apply dominance criteria in practice.

In conclusion, this study underscores the potential of stochastic dominance as a robust and theoretically grounded approach to risk management in banking. By capturing the full distribution of outcomes and accommodating diverse risk preferences, it provides valuable insights that complement traditional risk measures. While challenges remain in data requirements and computational complexity, the benefits of a more comprehensive risk evaluation suggest that stochastic dominance can play a significant role in enhancing both managerial decision-making and regulatory oversight in the banking sector (Embrechts et al., 2018).

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REFERENCES

Acerbi, C., & Tasche, D. (2002). On the Coherence of Expected Shortfall. *Journal of Banking & Finance*, 26(7), 1487–1503. https://doi.org/10.1016/S0378-4266(02)00283-2

Acharya, V. V., Engle, R., & Richardson, M. (2017). Measuring Systemic Risk. *The Review of Financial Studies*, 30(1), 2–47. https://doi.org/10.1093/rfs/hhw088

Basel Committee on Banking Supervision. (2011). Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems. Bank for International Settlements. https://www.bis.org/publ/bcbs189.htm

Basel Committee on Banking Supervision. (2019). Basel III: Finalizing Post-Crisis Reforms. Bank for International Settlements. https://www.bis.org/bcbs/publ/d424.htm

Brownlees, C., & Engle, R. F. (2017). SRISK: A Conditional Capital Shortfall Measure of Systemic Risk. *The Review of Financial Studies*, 30(1), 48–79. https://doi.org/10.1093/rfs/hhw060

Danielsson, J. (2011). Financial Risk Forecasting: The Theory and Practice of Forecasting Market Risk with Implementation in R and MATLAB. *Wiley*.

Embrechts, P., Wang, B., & Wang, R. (2018). Aggregation-Robust Risk Measures. *Finance and Stochastics*, 22(2), 433–461. https://doi.org/10.1007/s00780-015-0273-z

Hadar, J., & Russell, W. R. (1969). Rules for Ordering Uncertain Prospects. *American Economic Review*, 59(1), 25–34.

Hanoch, G., & Levy, H. (1969). The Efficiency Analysis of Choices Involving Risk. *The Review of Economic Studies*, 36(3), 335–346. https://doi.org/10.2307/2296431

Jorion, P. (2007). Value at Risk: The New Benchmark for Managing Financial Risk (3rd Ed.). *Mcgraw-Hill.*

Kou, S. G., & Peng, X. (2016). On the Measurement of Economic Tail Risk. Operations Research, 64(5), 1059–1070. https://doi.org/10.1287/opre.2016.1539

Levy, H. (2015). Stochastic Dominance: Investment Decision Making Under Uncertainty. *Springer.*

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77–91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x

Nwankwo, O., & Osho, G. S. (2010). An empirical analysis of corporate survival and growth: Evidence from efficient working capital management. *International Journal of Scholarly Academic Intellectual Diversity*, 12(1), 1–13.

Osho, G. S. (2008). How Technology is Breaking Traditional Barriers in the Banking Industry: Evidence From Financial Management Perspective. *European Journal of Economics, Finance and Administrative Sciences*, 11(3), 15–21.

Osho, G., Osagie, J., & Adams, M. O. (2005). Risk Management in Developing Countries: Evidence from Multinational and Country Financial Risk Analysis. Journal of Business & Economics Research, 3(8), 1–10. https://doi.org/10.19030/jber.v3i8.2800

von Neumann, J., & Morgenstern, O. (1944). Theory of Games and Economic Behavior. *Princeton University Press*.

https://www.jstor.org/stable/j.ctt1r2gkx

Appendixes

Appendix A: Mathematical Foundations and Proofs

This appendix presents the formal mathematical structure underlying stochastic dominance and its relationship to expected utility theory. Let $X$and Ydenote two random variables representing returns on banking assets with cumulative distribution functions $F_X(x)$and $F_Y(x)$. First-order stochastic dominance holds when $F_X(x) \leq F_Y(x)$for all x, with strict inequality for some values. This condition implies that for any non-decreasing utility function. $u(x)$, the expected utility satisfies

$$\mathbb{E}[u(X)] \geq \mathbb{E}[u(Y)] \tag{13}$$

Second-order stochastic dominance is established when

$$\int_{-\infty}^{x} F_X(t)\, dt \leq \int_{-\infty}^{x} F_Y(t)\, dt \forall x \tag{14}$$

which implies that for any increasing and concave utility function representing risk-averse behavior, the expected utility condition remains valid. The proof follows from integration by parts and the properties of concave functions as established in classical decision theory (Hadar & Russell, 1969; Hanoch & Levy, 1969).

Third-order stochastic dominance extends the framework by incorporating higher-order preferences associated with prudence. The condition

$$\int_{-\infty}^{x} \int_{-\infty}^{s} F_X(t)\, dt\, ds \leq \int_{-\infty}^{x} \int_{-\infty}^{s} F_Y(t)\, dt\, ds \tag{15}$$

ensures that decision makers with decreasing absolute risk aversion will prefer $X$to Y. These proofs collectively demonstrate that stochastic dominance provides a consistent ordering of risky prospects without requiring specification of a particular utility function (Levy, 2015).

Appendix B: Additional Tables and Figures

This appendix provides supplementary empirical results that support the main analysis presented in the study. Table B1 reports extended descriptive statistics, including higher moment measures and percentile values for the key variables.

Table B1. Extended Descriptive Statistics

Variable	5th Percentile	25th Percentile	75th Percentile	95th Percentile
Portfolio Return	-0.085	0.045	0.118	0.175
Default Probability	0.015	0.042	0.089	0.142
Loss Given Default	0.210	0.330	0.520	0.690

Figure B1 illustrates the empirical cumulative distribution functions of selected portfolios. The graphical representation highlights the regions where stochastic dominance relationships hold, particularly under second-order conditions.

Figure B1. Empirical Cumulative Distribution Functions of Portfolios

The curves show that the conservative portfolio lies below the others across a broad range of outcomes, indicating lower cumulative loss probabilities and supporting the dominance results discussed in the main text.

Table B2 presents additional regression results that examine the sensitivity of portfolio returns to alternative risk measures.

Table B2. Extended Regression Analysis

Variable	Coefficient	Std. Error	t Statistic
VaR	-0.298	0.079	-3.772
Expected Shortfall	-0.415	0.091	-4.560
Skewness	0.126	0.048	2.625

The inclusion of skewness underscores the importance of higher-order distributional properties in explaining portfolio performance, further justifying the use of stochastic dominance.

Appendix C: Model Derivations

This appendix details the derivation of the models used in the empirical analysis. Portfolio returns are constructed as weighted sums of individual asset returns.

$$R_p = \sum_{i=1}^{n} w_i R_i \tag{16}$$

subject to the constraint $\sum_{i=1}^{n} w_i = 1$. The distribution of portfolio returns is derived from the joint distribution of asset returns, which may incorporate correlations among assets.

The empirical cumulative distribution function is derived as

$$\hat{F}_n(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(R_i \leq x) \tag{17}$$

which converges to the true distribution function as the sample size increases.

Value at Risk is derived as the quantile of the distribution such that.

$$\mathrm{VaR}_\alpha = \inf\{x: F(x) \geq \alpha\} \tag{18}$$

while Expected Shortfall is defined as the conditional expectation of losses beyond this threshold

$$\mathrm{ES}_\alpha = \frac{1}{\alpha}\int_0^\alpha \mathrm{VaR}_u \, du \tag{19}$$

These derivations provide the foundation for comparing traditional risk measures with stochastic dominance criteria. By integrating these models within a unified analytical framework, the study establishes a rigorous basis for evaluating risk in banking systems.